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LMU and TU München
Sommersemester 2017

Algebraic Quantum Field Theory
Homework Sheet 1

Problem 1. Let $\mathcal{H}_1 = L^2(\mathbb{R}^n)$ with the scalar product $\langle f, g\rangle = \int d^n x\, \overline{f}(x) g(x)$. Show that the prescription

$$\big(\pi_1(W(z))f\big)(x) = e^{\frac{i}{2}uv} e^{ivx} f(x+u), \quad z = u + iv. \tag{1}$$

defines a representation of $\mathcal{W}$.

Problem 2. Let $\mathcal{H}_2 = L^2(\mathbb{R}^n)$ with the scalar product $\langle f, g\rangle = \int d^n x\, \overline{f}(x) g(x)$. One defines

$$\big(\pi_2(W(z))f\big)(x) = e^{-\frac{i}{2}uv} e^{iux} f(x-v), \quad z = u + iv. \tag{2}$$

Show that this prescription defines a representation of $\mathcal{W}$. Also show that this representation is unitarily equivalent to the representation from Exercise 1 with the Fourier transform being the unitary.

Problem 3. Show that there is no representation of $\mathcal{W}$ on a finite dimensional Hilbert space (apart from the trivial one $\pi(W) = 0$ for all $W \in \mathcal{W}$). Hints

(i) First check that you can assume without loss of generality that $\pi(1) = 1$.

(ii) Next use properties of the determinant and the relation

$$W(z)W(z')W(z)^* = e^{i\mathrm{Im}\langle z, z'\rangle} W(z'). \tag{3}$$

Problem 4. Show that the representation from Problem 1 is irreducible. Hints:

(i) It suffices to show that given $f, g \in L^2(\mathbb{R}^n)$, $f \neq 0$, the equality

$$\langle g, \pi_1(W(z))f\rangle = 0 \tag{4}$$

for all z implies $g = 0$. (This implies that f is cyclic for $\pi_1(\mathcal{W})$).

(ii) Use that the Fourier transform is injective on $L^1(\mathbb{R}^n)$.

To be discussed in class: Thursday 8.5.2017