3. Proof of Compression Theorem ⇒ Game Value a.k.a. Halting

<u>Theorem</u> (Compression theorem, version 2.0)

There exists a universal constant $C_0 > 0$ s.t. the following holds:
There exists a polynomial-time Turing machine COMPRESS that takes as input a NFV $\mathcal{V} = (\mathcal{S}, \mathcal{D})$ and $\lambda > 0$ integer, and returns the description of a 9-level NFV $\mathcal{V}^{\text{COMPR}} = (\mathcal{S}^{\text{COMPR}}, \mathcal{D}^{\text{COMPR}})$ with:

$$\text{TIME}_{\mathcal{S}^{\text{COMPR}}}(n), \text{TIME}_{\mathcal{D}^{\text{COMPR}}}(n) = \text{poly}(n, \lambda).$$

Moreover, $|\mathcal{D}^{\text{COMPR}}| = \text{poly}(|\mathcal{V}|, \log \lambda)$, $|\mathcal{S}^{\text{COMPR}}| = \text{poly} \log \lambda$.

If $\mathcal{V}$ is 9-level and λ-bounded, $\forall n \geq C_0$,

1) If $\mathcal{V}_{2^n}$ has a value-1 PCC strategy $\Rightarrow$ $\mathcal{V}_n^{\text{COMPR}}$ does.

2) If $\text{val}^*(\mathcal{V}_{2^n}) \leq 1/2 \Rightarrow \text{val}^*(\mathcal{V}_n^{\text{COMPR}}) \leq 1/2$.

3) $\mathcal{E}(\mathcal{V}_n^{\text{COMPR}}, 1/2) \geq \max\{\mathcal{E}(\mathcal{V}_{2^n}, \frac{1}{2}), 2^{2^{\Omega(n)}}\}$.