Infinite hatted queue Paradox

Finite Prisoners and Hats Puzzle

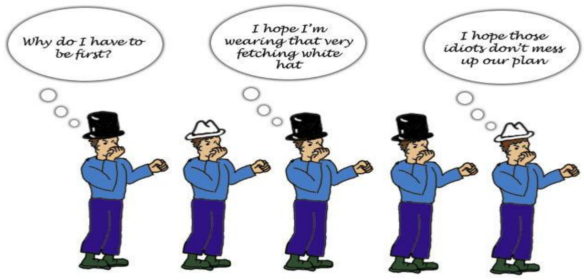
- A finite number of Prisoners are placed in a line facing forward
- A warden places either a black or a white hat on the prisoners head without them seeing which they have on their head
- Starting at the back the warden asks the prisoners one by one what color hat they have on their head
- If the prisoner answers right then he is released, if wrong then he is executed ☺
- The prisoners hear each other's answers and whether these were correct or not
- The prisoners are given time to come up with a strategy before the game begins, what is the best strategy?

Solution

The prisoners agree on a "code" beforehand. They agree that white represents 'even' and black 'odd' so now the first prisoner simply has to look in front of him and count the number of white hats he sees. If this number is odd then he says black and if even then white (as agreed prior). Hence the second prisoner in line knows if the previous prisoner saw an odd or even number of white hats, all the second prisoner is left to do is count the number of white hats himself and thus determine his own color. This goes on until the last person also guesses his hat color correctly.

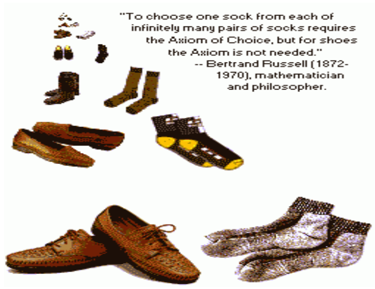
Scenario for the infinite case

- Countable infinite number of prisoners
- Prisoners cannot hear each other



Axiom of Choice

For every indexed family of non empty sets there exists a function which chooses an element from each set.



Problems with this definition: The Axiom does not specify what this function is, nor any construction of it. It only declares its existence.

Solution to the infinite case:

Substitute the colors by numbers and get an infinite sequence of numbers. Define an equivalence relation, by stating that 2 sequences are equivalent, if they are equal after a finite number of entries. **Invoke the Axiom of Choice** to pick a representative out of each equivalence class. Then all prisoners memorize the same representative of each class and once they are placed in line they identify the equivalence class they are in by looking the prisoners ahead. Now they guess as if they were in the pre-chosen element in that equivalence class. Hence after a finite number of incorrect guesses, each prisoner will guess his color correctly!

Banach Tarski Paradox:

Given a solid ball in the 3-dimensional space, there exists a decomposition of that ball into a finite number of disjoint subsets, which can be rearranged by translations and rotations to result in two identical copies of the original ball.

"A pea can be chopped up and reassembled into the Sun"

Summary of important steps needed to complete proof:

- Find a paradoxical decomposition of the group of Rotations on two rotation axis (two generators)
- Use this decomposition and the Axiom of Choice to perform a paradoxical decomposition of the empty unit sphere (Hausdorff paradox). This can then be extended to the solid sphere which results in the full Banach Tarski Paradox.

Bibliography:

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- http://en.wikipedia.org/wiki/Prisoners_and_hats_puzzle
- http://en.wikipedia.org/wiki/Banach Tarski_paradox
- http://en.wikipedia.org/wiki/Axiom_of_choice
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