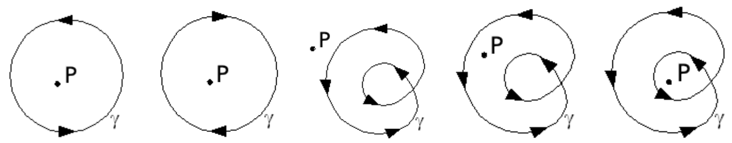
1 Inside and Outside of Loops & Knots

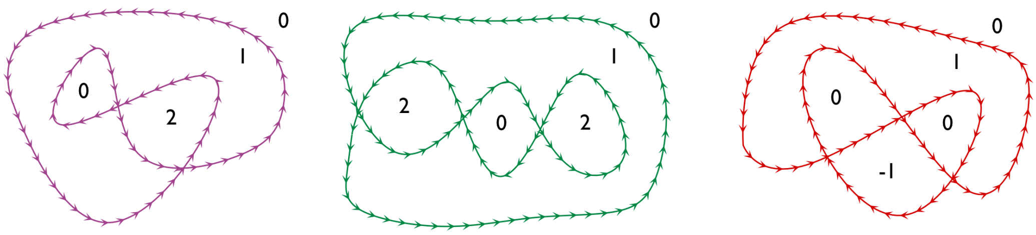
1.1 'Inside-Outside' Knot Trick

Winding Number The winding number $Ind_\gamma(P)$ of a closed curve γ in the plane around a given point P is an integer representing the total number of times that curve travels counterclockwise around the point. If $Ind_\gamma(P) = 0$ then P is *outside*, otherwise P is *inside*.
If the curve travels around the point clockwise, the winding number is negative [1].



(a)	(b)	(c)	(d)	(e)
$Ind_\gamma(P)$=1	$Ind_\gamma(P)$=-1	$Ind_\gamma(P)$=0	$Ind_\gamma(P)$=1	$Ind_\gamma(P)$=2

Game [2]: Place your hand inside ⇒ WIN Place your hand outside ⇒ LOSE



(a) Fair Throw (b) Superfair Throw (c) Cheat Throw

(a) 0:lose 2:win 1:*invisible* (b) 2:win 1,0:*invisible* (c) 0:lose 1,-1:*invisible*

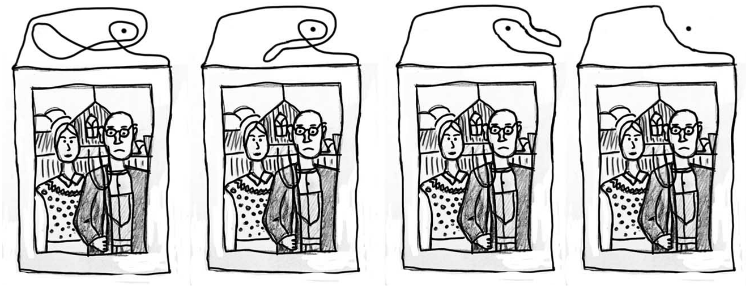
1.2 Picture Hanging Problem

Definition The *Picture Hanging Problem* asks if it is possible to hang a picture on n nails in such a way that when one nail is removed, the picture falls to the ground.

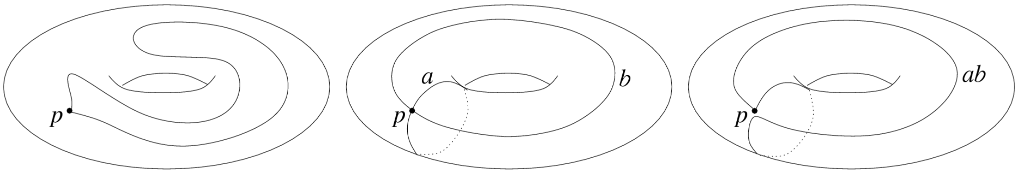
Constraints of the Picture Hanging Problem

- The loop must loop around each of the n nails.
- If any nail is removed, the loop should be reducible to a loop around none of the nails.

Fundamental Group The fundamental group of a topological space X is the set of equivalence classes of loops about a basepoint p, under the equivalence relation of homotopy, with the group operation of loop composition. We will denote such a group $\pi_1(X, p)$ or $\pi_1(X)$.
The neutral element is the (homotopy) class of loops which can be contracted to the basepoint.
The inverse element to a class of loops is the class obtained by covering the loops backwards.
For $\beta_i \in Base(\pi_1)$: β_i corresponds to a counterclockwise loop, β_i^{-1} to a clockwise loop.



Example for $n = 2$ [3]



(a) contractible loop (b) 2 non contractible loops (c) composition of 2 loops

Example: Torus T^2 $\pi_1(T^2) = \mathbb{Z}^2$ [4]

Base of the Fundamental Group of G_n, the n-punctured plane Base(G_n)=$\{\beta_1, ..., \beta_n\}$

Solution of the Picture Hanging Problem: $\forall n \in \mathbb{N}: \quad l_n = l_{n-1}\beta_n^{-1}(l_{n-1})^{-1}\beta_n$

n=1 $l_1 = \beta_1$

n=2 $l_2 = \beta_1\beta_2^{-1}\beta_1^{-1}\beta_2$

n=3 $l_3 = \beta_1\beta_2^{-1}\beta_1^{-1}\beta_2\beta_3^{-1}\beta_2^{-1}\beta_1\beta_2\beta_1^{-1}\beta_3$

n=4 $l_4 = \beta_1\beta_2^{-1}\beta_1^{-1}\beta_2\beta_3^{-1}\beta_2^{-1}\beta_1\beta_2\beta_1^{-1}\beta_3\beta_4^{-1}\beta_3^{-1}\beta_1\beta_2^{-1}\beta_1^{-1}\beta_2\beta_3\beta_2^{-1}\beta_1\beta_2\beta_1^{-1}\beta_4$

References

[1] http://de.wikipedia.org/wiki/Windungszahl

[2] P. Diaconis und R. Graham. Magical Mathematics: The Mathematical Ideas that Animate Great Magic Tricks. (2011) , 164-171.

[3] http://strangenewuniverse.wordpress.com/2012/04/26/homology-groups/

[4] http://de.wikipedia.org/wiki/Fundamentalgruppe