

1. Intro

GRS-model (Gilbert/Shannon/Reeds):

- Chance that k cards are cut off from n cards: $\binom{n}{k}/2^n$
- Chance that the next card will drop from the left heap : $A/(A+B)$

2. Card trick: "Premo" (Jordan)

Options:

- Cut the deck, riffle shuffle once, cut again, riffle shuffle again, remove the top card and insert it
- Move the card after the last shuffle or diversify the number of shuffles.
- "Long-distance mindreading"

Important: rising sequences, winding number

Monte Carlo – Experiments:

Probability of success in Jordan's card trick with 52 cards shuffled m times, and 1, 2, 3, 13 or 26 guesses allowed. Each entry is based on 1,000,000 Monte Carlo trials. All entries are given in thousandths

m	2	3	4	5	6	7	8	9	10	11	12	∞	m = # shuffles
1	997	839	288	088	042	028	023	021	020	020	019	019	
2	1000	943	471	168	083	057	047	042	040	039	039	038	
3	1000	965	590	238	123	085	070	063	061	059	058	058	
13	1000	998	884	617	427	334	290	270	260	254	252	250	
26	1000	999	975	835	688	596	548	524	513	505	503	500	

3. Shuffles and their generalizations

Generalization : a = # packets

Theorem: Probability that an a-shuffle results in permutation π : $\binom{a+n-r}{n}/a^n$

4. The approach to uniformity

We derive approximations when n is large after m shuffles with $m = \frac{3}{2} \log_2 n + \theta$

for $n = 52 \rightarrow m \approx 8.55$

Total variation distance: $\|Q^m - U\| = \max_{A \subset S_n} |Q^m(A) - U(A)|$

$$Q^m = \binom{2^m + n - r}{n} / 2^{mn} \text{ GRS probability after m 2-shuffles,}$$

U = uniform probability: $U(\pi) = 1/n!$, S_n = symmetric group,

Theorem: $\|Q^m - U\| = 1 - 2 \Phi\left(\frac{-2^{-\theta}}{4\sqrt{3}}\right) + O\left(\frac{1}{n^{1/4}}\right)$, $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$

Source:

Bayer, Diaconis: Trailing the dovetail shuffle to its lair (1992)

Diaconis, Graham: Magical Mathematics (2011, pages 189-200)

<https://sites.google.com/site/erfarmer/premo>

<http://possiblywrong.wordpress.com/2011/04/04/analysis-and-a-variant-of-the-premo-card-trick/>