TECHNISCHE UNIVERSITÄT MÜNCHEN
Zentrum Mathematik

Dr. David Reeb

Mathematical Basis of Quantum Statistical Physics

http://www-m5.ma.tum.de/Allgemeines/MA5109_2013W

WS 2013/14
Blatt 0
(14.10.2013)

This sheet is meant to give me some information about your background. You are *not* supposed to be able to answer all questions. Please indicate the questions with which you are not at all familiar.

0.1. Functional analysis: operator norm

(a) Define $L^\infty(\mathbb{R})$.

(b) Let $A : \mathcal{H} \to \mathcal{H}$ be a bounded linear operator on a Hilbert space $\mathcal{H}$. Define/characterize the operator norm $\|A\|_\infty$ (sometimes written just as $\|A\|$).

(c) Compute $\|A\|_\infty$ for the matrix $A := \begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix}$ (which is written with respect to an orthonormal basis in $\mathbb{C}^2$).

0.2. Functional analysis: different notions of convergence

For a Hilbert space $\mathcal{H}$, let A and A_n (for $n \geq 1$) be bounded linear operators $A : \mathcal{H} \to \mathcal{H}$, $A_n : \mathcal{H} \to \mathcal{H}$. Write down mathematical versions of the following three statements:

(a) The sequence $(A_n)_{n\geq 0}$ converges to A in the uniform operator topology (i.e. converges uniformly; converges in norm).

(b) $(A_n)_{n\geq 0}$ converges to A in the strong operator topology (i.e. converges strongly).

(c) $(A_n)_{n\geq 0}$ converges to A in the weak operator topology (i.e. converges weakly).

0.3. Quantum mechanics: time-evolution

(a) Write down the Schrödinger equation.

(b) Write down Heisenberg's equation of motion for an obervable A subject to a time-independent Hamiltonian H. What's the solution of this equation?

0.4. Bra-ket notation vs. vector notation; "GNS setting"

Which of the following two formulations do you prefer?

(i) Let $\mathcal{H}_A := \mathcal{H}_B := \mathbb{C}^3$ be Hilbert spaces with orthonormal basis $\{|1\rangle, |2\rangle, |3\rangle\}$, and define $|\psi\rangle := \frac{4}{9}|1\rangle|1\rangle + \frac{1}{9}|2\rangle|2\rangle + \frac{8}{9}|3\rangle|3\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$, and $\rho_{AB} := |\psi\rangle\langle\psi| \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B)$. What are the eigenvalues of ρ_{AB}? What are the eigenvalues of the reduced state $\rho_A := \mathrm{tr}_B[\rho_{AB}]$? Write down the state ρ_A (e.g. in matrix form).

(ii) Let $\mathcal{H}_A := \mathcal{H}_B := \mathbb{C}^3$ be Hilbert spaces with orthonormal basis $\{\psi_1, \psi_2, \psi_3\}$, and define $\psi := \frac{4}{9}\psi_1 \otimes \psi_1 + \frac{1}{9}\psi_2 \otimes \psi_2 + \frac{8}{9}\psi_3 \otimes \psi_3 \in \mathcal{H}_A \otimes \mathcal{H}_B$, and $\rho_{AB} := \psi\psi^* \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B)$. What are the eigenvalues of ρ_{AB}? What are the eigenvalues of the reduced state $\rho_A := \mathrm{tr}_B[\rho_{AB}]$? Write down the state ρ_A (e.g. in matrix form).

(a) Your preference: *(i)* or *(ii)* . You *have* heard of bra's and ket's before: *yes* *no*

(b) Answer the above questions in your favourite language!

0.5. Statistical Mechanics

(a) For a Hamiltonian H and a temperature T (or "inverse temperature" β), what is the corresponding thermal state ρ_{th} ("Gibbs state")? What is the partition function?

(b) What is the entropy and energy of ρ_{th}?

0.6. Matrix computations

(a) Is the matrix $\rho := \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$ positive semidefinite?

(b) Compute $\mathrm{tr}\,[\rho \log \rho]$.