$\boxed{\text{Blatt 7}}$

1. a) $T^*(A) = \lambda A + (1-\lambda)\, tr[A\sigma]\, \mathbb{1}$

b) $T^*: \mathcal{B}(\mathcal{H}_1) \to \mathcal{B}(\mathcal{H}_1 \otimes \mathcal{H}_2)$

$T^*(A) = A \otimes \mathbb{1}$

c) $tr[T(\varrho)\, A] = tr[\varrho \otimes \sigma\, A]$

$$\overset{!}{=} tr[\varrho\, T^*(A)] \qquad = tr[(\varrho \otimes \mathbb{1})(\mathbb{1} \otimes \sigma) A]$$

$$= tr_1[\varrho\, tr_2[(\mathbb{1}\otimes\sigma) A]]$$

$\longrightarrow \quad T^*(A) = tr_2[(\mathbb{1}\otimes\sigma) A]$

d) $T^*(A) = (d-1)^{-1}\left(tr[A]\, \mathbb{1} + A^T\right)$

2. $\Sigma_1 := \sigma_1 \otimes \mathbb{1}_2 + \sigma_2 \otimes \sigma_1$

$\Sigma_2 := \sigma_2 \otimes \mathbb{1}_2 + \sigma_1 \otimes \sigma_1$

3. Let $H_1, \dots, H_{d^2}$ be a Hermitian ONB in $\mathcal{B}(\mathbb{C}^d)$ and $K_1, \dots, K_{d^2} \in \mathcal{B}(K)$ with $\dim(K) = (n+1)d$ commuting Hermitian extensions s.t. $H_i = V^* K_i V$ for some isometry $V: \mathcal{H} \to K$. For $\varrho = \sum_i c_i H_i$ then set $R(\varrho) := \sum_i c_i K_i$.

4. Let $k \in \mathbb{N}$ be that number. Clearly, $k \geq 2$ since otherwise T would be $T(\cdot) = U \cdot U^*$ for some unitary U. However, $k=2$ is feasible: $T(\varrho) = p\varrho + (1-p)\, \sigma_3 \varrho \sigma_3$ with $p = 1 - \frac{\lambda}{2}$.