TECHNISCHE UNIVERSITÄT MÜNCHEN
Zentrum Mathematik

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Mathematical Introduction to Quantum Information Processing

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http://www-m5.ma.tum.de/Allgemeines/MA5057_2019S

Read the section *Quantum channels and operations* in the lecture notes!

1. Consider finite-dimensional Hilbert spaces.
   (a) Show that any linear map $T : \mathcal{B}(\mathcal{H}_1) \to \mathcal{B}(\mathcal{H}_2)$ can be written as a linear combination of four completely positive maps.
   (b) Write matrix transposition $\Theta(A) := A^T$ as a real linear combination of two completely positive maps.
   (c) Use the definition of complete positivity to prove that $X \to AXA^*$ is completely positive for any $A \in \mathcal{B}(\mathcal{H}_1, \mathcal{H}_2)$.
   (d) Show that if T_1, T_2 are completely positive maps, then $T_1 \circ T_2$, $T_1 + T_2$, $T_1 \otimes T_2$ are completely positive as well.
2. How can one visualize trace-preserving positive maps $T : \mathbb{C}^{2\times 2} \to \mathbb{C}^{2\times 2}$ in the Bloch-ball representation?
3. Let $K \in \mathbb{C}^{d\times d}$ be such that $K^T = -K$ and $K^*K \leq \mathbb{1}$. Show that the map $T : \mathbb{C}^{d\times d} \to \mathbb{C}^{d\times d}$ defined as $T(X) := \mathrm{tr}\,[X]\,\mathbb{1} - X - KX^TK^*$ is positive. Is it completely positive?
4. Decoherence and decay processes can often be described by a map of the form

$$T(\rho) = e^{-t}\rho + \left(1 - e^{-t}\right)\mathrm{tr}\,[\rho]\,\sigma,$$

   where $t \in \mathbb{R}_+$ and σ is a density operator. Find a Kraus representation for this map.