TECHNISCHE UNIVERSITÄT MÜNCHEN

Zentrum Mathematik

Prof. Dr. M. Wolf **Mathematical Introduction to Quantum Information Processing**

Summer 2019

Blatt 4

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http://www-m5.ma.tum.de/Allgemeines/MA5057_2019S

Read the section *Convexity* in the lecture notes!

1. Show that two preparations described by density operators $\rho_1, \rho_2 \in \mathcal{B}_1(\mathcal{H})$ can be distinguished with certainty in a statistical experiment iff $\rho_1\rho_2 = 0$.
2. Construct a pair of density operators ρ, ρ' on a common Hilbert space with the properties that: (i) their spectra coincide and each eigenvalue has multiplicity one, (ii) there is no unitary U such that $U\rho U^* = \rho'$.
3. Show that pure states are extreme points of the convex set of density operators.
4. Let $\rho_1, \rho_2 \in \mathcal{B}(\mathbb{C}^d)$ be two density operators. Prove that $\rho_1 \prec \rho_2$ iff there exist a finite set of unitaries $U_i \in \mathcal{B}(\mathbb{C}^d)$ and corresponding probabilities $p_i > 0$, $\sum_i p_i = 1$ so that $\rho_1 = \sum_i p_i U_i \rho_2 U_i^*$.
5. Construct a sequence of density operators of finite entropy that converges in trace-norm to a pure state but has entropy diverging to ∞.