Quantum Effects [MA5047]

Sheet 7

Discussion: 04.02.2014

Exercise 1: Show that there exists a density operator $\rho_{ABC} \in \mathcal{B}(\mathbb{C}^d \otimes \mathbb{C}^d \otimes \mathbb{C}^d)$ such that the reduced density matrices fulfill $\rho_{AB} = \rho_{AC}$ and given $|\Omega\rangle := \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |i,i\rangle$ the maximally entangled state, it holds

$$\langle\Omega|\rho_{AB}|\Omega\rangle \geq \frac{1}{2} + \frac{1}{2d^2}$$

(Hint: You might want to consider nonsymmetrized states first, i.e. where $\rho_{AB} \neq \rho_{AC}$, and then symmetrize).

Exercise 2: Let $\mathbb{F}$ denote the flip operator (i.e. $\mathbb{F}(|i,j\rangle) = |j,i\rangle$ on $\mathbb{C}^d \otimes \mathbb{C}^d$). Show:

1. If ρ is separable, then $\mathrm{tr}(\rho\mathbb{F}) \geq 0$.
2. Let $P_- \in \mathcal{B}(\mathbb{C}^d \otimes \mathbb{C}^d)$ be the projection onto the anti-symmetric subspace $\mathcal{H}_- := \{\psi | \mathbb{F}\psi = -\psi\}$. Show that $\rho := \frac{P_-}{\mathrm{tr}(P_-)}$ is entangled.

Exercise 3: (unitary freedom of Kraus operators) Let $T : \mathcal{B}(\mathbb{C}^d) \to \mathcal{B}(\mathbb{C}^d)$ be a quantum channel with Kraus operators $\{E_1, \dots, E_m\}$. Show that another set of Kraus operators $\{F_1, \dots, F_m\}$ equivalently describes the quantum channel, if and only if after appending the shorter list of operators with zeroes such that $m = n$ there exists a unitary $U \in \mathcal{C}^{n\times n}$ with $E_i = \sum_j U_{ij} F_j$.

Exercise 4: (quantum steering) Let $\rho \in \mathcal{B}(\mathbb{C}^d)$ be a density operator with purification $|\psi\rangle \in \mathcal{C}^d \otimes \mathcal{C}^d$. For every convex decomposition $\rho = \sum_i \lambda_i \rho_i$ there is an instrument $\{T_i : \mathcal{B}(\mathbb{C}^d \to \mathbb{C}^d)\}$ acting on the second system such that:

$$\lambda_i \rho_i = \mathrm{tr}_2[(\mathbb{1} \otimes T_i)(|\psi\rangle\langle\psi|)]$$

Hint: Recall the results from sheet 3 exercise 4. In part 2 of that exercise, it might be interesting to construct an explicit R.