Quantum Effects [MA5047]

Sheet 4

Discussion: 10.12.2013

Exercise 1: Let $\mathcal{C} \subseteq \mathbb{R}^{m\times m}$ be the set of matrices C for which there are random variables $A_x, B_y : \Omega \to \{-1, 1\}$ and a probability measure P such that

$$C_{xy} = \int_\Omega A_x(\omega) B_y(\omega)\, \mathrm{d}P(\omega)$$

1. Show that $\mathcal{C}$ is a closed, convex polytope.
2. Let $\mathcal{C}' \subseteq \mathbb{R}^{m\times m}$ be a similarly defined set for which the random variables are allowed to have ranges in $[-1, 1]$ rather than only $\{-1, 1\}$. Prove that $\mathcal{C}' = \mathcal{C}$.

Exercise 2: Consider a setup of three parties (Alice, Bob and Charly), each of which has two ± 1-valued measurement devices at hand, which we label by P_i, where the $P \in \{A(lice), B(ob), C(harly)\}$ stands for the party and the $i \in \{1, 2\}$ for the chosen device. For

$$\langle A_1 B_1 C_2 \rangle + \langle A_1 B_2 C_1 \rangle + \langle A_2 B_1 C_1 \rangle - \langle A_2 B_2 C_2 \rangle$$

quantum theory allows a value up to 4. What is the maximum value consistent with a LHV theory (generalized to three parties)?

Exercise 3: Let $\rho \in \mathcal{B}(\mathcal{H})$ be a density operator such that $\ker \rho = \{0\}$. Define $\langle \cdot, \cdot \rangle_\rho : \mathcal{B}(\mathcal{H}) \times \mathcal{B}(\mathcal{H}) \to \mathbb{C}$ by $\langle A, B \rangle_\rho := \mathrm{tr}[\rho A^* B]$.

- Prove that $\langle \cdot, \cdot \rangle_\rho$ is a scalar product.
- Prove that $\mathrm{tr}[\rho^2]^2 \leq \mathrm{tr}[\rho^3]$