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Asymptotic Geometric Analysis

http://www-m5.ma.tum.de/Allgemeines/MA5024_2015W

Sommers. 2015
Blatt 1
(29.04.2015)

Hausaufgaben

1.1. Repetition

a) What is an extreme point of a convex set? Determine the extreme points of $B_{l_1^2}$ and $B_{(\mathcal{B}(l_2^n,\ l_2^n),\|\cdot\|_{op})}$.

b) What is a simplex?

c) What is Carathéodory's theorem?

1.2. Polars and minimal ellipsoids

Let $X \simeq \mathbb{R}^n$ be a Banach space with unit ball B_X.

a) Let $K, L \subseteq X$ be convex sets such that $K \subseteq L$. Show that $L^\circ \subseteq K^\circ$. What does this imply for the unit balls of the duals of Banach spaces X and Y if $B_X \subseteq B_Y$?

b) Show that $(L \cup K)^\circ = L^\circ \cap K^\circ$.

c) Let $K \subseteq X$ be a convex symmetric body. Use (a) to show that there exists a unique ellipsoid $E \subseteq X$ of minimal volume such that $K \subseteq E$. You are allowed to use that there always is a unique maximum volume ellipsoid inside a given convex symmetric body (this will be proven in the lecture). Here 'ellipsoid' always refers to the image of the l_2^n unit ball under an isomorphism.

d) Let $E \subseteq X$ be an ellipsoid. Show that

$$\mathrm{vol}(E)\mathrm{vol}(E^\circ) = \mathrm{vol}(B_{l_2^n})^2.$$

e) Assume $B_{l_2^n} \subseteq B_X$. Furthermore, assume that there exist $\{u_1, ..., u_N\} \subseteq \partial B_{l_2^n} \cup \partial B_X$ such that $\mathbb{1} = \sum_{i=1}^N \lambda_i u_i u_i^*$ for some $\lambda \in \mathbb{R}_+^N$. Prove that $B_{l_2^n}$ is the maximum volume ellipsoid contained in B_X.

f) Show that $B_{l_2^n}$ is the maximum volume ellipsoid contained in $B_{l_\infty^n}$.

g) What is the minimum volume ellipsoid that contains $B_{l_1^n}$? Give a proof.