



The following homework will be discussed in the first week of July:

I. Kernels - basic properties

1. Let $K : X \times X \rightarrow \mathbb{R}$ be a PSD-Kernel. Prove that for all $x, y \in X$: $|K(x, y)|^2 \leq K(x, x)K(y, y)$.
2. Let $K_n : X \times X \rightarrow \mathbb{R}$ be a sequence of PSD-Kernels. Prove that the pointwise limit $K := \lim_{n \rightarrow \infty} K_n$ is again a PSD Kernel.
3. For an PSD Kernel $K : X \times X \rightarrow \mathbb{R}$ define

$$K'(x, y) := \begin{cases} 0, & \text{if } K(x, x)K(y, y) = 0 \\ \frac{K(x, y)}{\sqrt{K(x, x)K(y, y)}} & \text{otherwise.} \end{cases} \quad (1)$$

Show that K' is a PSD Kernel, as well.

II. PSD Kernels - examples

Prove or disprove that the following are PSD Kernels (all norms are assumed Euclidean):

1. $X := \{x \in \mathbb{R}^d \mid \|x\| < 1\}$, $K : X \times X \rightarrow \mathbb{R}$, $K(x, y) := (1 - \langle x, y \rangle)^{-1}$.
2. $K : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R}$, $K(x, y) := \sinh(\langle x, y \rangle^4)$.
3. $K : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R}$, $K(x, y) := \|x - y\|^2$.