TECHNISCHE UNIVERSITÄT MÜNCHEN
Zentrum Mathematik

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Mathematical Foundations of Machine Learning

http://www-m5.ma.tum.de/Allgemeines/MA4801_2016S

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Blatt 3
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The following homework will be discussed on May 11th to 13th:

I. Growth function

1. Consider function classes $\mathcal{F}_1 \subseteq \mathcal{Y}^{\mathcal{X}}, \mathcal{F}_2 \subseteq \mathcal{Z}^{\mathcal{Y}}$ and $\mathcal{F} := \mathcal{F}_2 \circ \mathcal{F}_1$. Let Γ_1, Γ_2 and Γ be the respective growth functions. Provide an upper bound on Γ in terms of Γ_1 and Γ_2.

2. Compute the growth function of the set $\mathcal{F} := \{x \mapsto \mathbb{1}_{x\in[a,b]}\}_{a,b\in\mathbb{R}} \subseteq \{0,1\}^{\mathbb{R}}$.

II. VC-dimension

For any set $\mathcal{C}$ of Borel subsets of $\mathbb{R}^d$ define $\mathcal{F} := \{x \mapsto \mathbb{1}_{x\in C}\}_{C\in\mathcal{C}}$ the class of corresponding indicator functions on $\mathbb{R}^d$ and set $\text{VCdim}(\mathcal{C}) := \text{VCdim}(\mathcal{F})$.

1. For any two function classes that satisfy $\mathcal{F}_1 \subseteq \mathcal{F}_2$ show that $\text{VCdim}(\mathcal{F}_1) \leq \text{VCdim}(\mathcal{F}_2)$.

2. Compute the VC-dimension of the set of all rectangular, axes-parallel boxes in $\mathbb{R}^d$. That is, all sets of the form $[a_1, b_1] \times \ldots \times [a_d, b_d]$. (Hint: start with $d = 2$, argue graphically by considering the smallest box containing all considered points.)

3. Let $\mathcal{C}$ be the class of all closed balls in $\mathbb{R}^d$, i.e., sets of the form $C := \{x \in \mathbb{R}^d | \; ||x-v||_2 \leq r\}$. Show that $\text{VCdim}(\mathcal{C}) \leq d + 2$.

4. Show that the VC-dimension of $\mathcal{F} := \{x \mapsto \text{sgn}[\sin(\phi x)]\}_{\phi\in\mathbb{R}} \subseteq \{-1,1\}^{\mathbb{R}}$ is infinite.

5. Take $\mathcal{F}$ from 3. and construct $A \subseteq \mathbb{R}$ with $|A| = 4$ so that $\mathcal{F}|_A \neq \{-1,1\}^A$.