# Entanglement Entropy and Algebraic Holography

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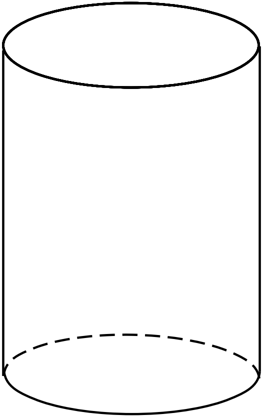
# The AdS/CFT correspondence (Maldacena et al 1998-)

is a relation between quantum gravity (string theory) in $d+2$ dimensional AdS and a CFT on the $d+1$ dimensional conformal boundary.

Picture $(d=1)$

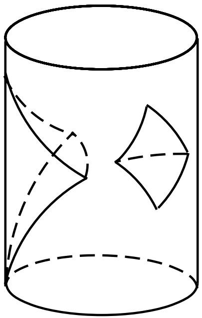


# Algebraic holograpy (Rehren 2000)

[**Note** that this is unrelated to quantum gravity and **note** that it is a theorem!]

A CFT on the conformal boundary is equivalent to a QFT on a fixed AdS background.

Key idea: double cone on boundary $\leftrightarrow$ wedge in AdS bulk



# The Arnsdorf-Smolin puzzle (2001)

– see discussion in BS Kay and L Ortiz: Brick Walls and AdS/CFT (2014)

If CFT $\equiv$ Quantum Gravity and CFT $\equiv$ QFTCST,

then Quantum Gravity $\equiv$ QFTCST.

Not reasonable!

# A way out?

(I'm not aware of any counter-evidence and am aware of evidence from several statements in the string-theory literature also I have my own arguments based on my (1998-2016) matter gravity entanglement hypothesis):

CFT $\equiv$ Just the matter sector of Quantum Gravity.

**Also:** The physical interpretation of the Rehren bulk QFT is that[1] it's an approximate description of the matter sector of quantum gravity.

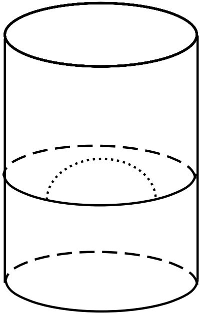
The purpose of the rest of the talk is to give a further piece of evidence for this based on consideration of Rehren's algebraic holography combined with the Ryu-Takayanagi entanglement equality.

[1](say in $d = 1$, assuming the Brown-Henneaux relation $c = 3/2G\sqrt{-\Lambda}$ holds)

# The Ryu-Takayanagi (RT) Entanglement Equality (2006)

[**Note:** The statement below only really makes sense in the presence of a suitable cut-off, otherwise both sides of the equality are infinite! See RT.]

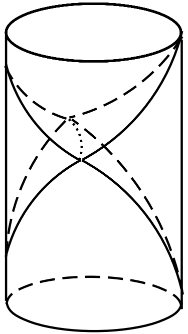
The entanglement entropy between two complementary intervals (in $d > 1$, complementary balls) on a fixed-time circle (in $d > 1$, sphere) of the conformal boundary is[1] (see Footnote on previous page) equal to $1/4G$ times the length of the geodesic (in $d > 1$, area of the minimal surface) which reaches the conformal boundary at the junction of the intervals.



# An entanglement equality for the Rehren bulk theory

### Easy geometry theorem

*The RT-geodesic (in $d > 1$, RT-minimal surface) is the shared ridge of the complementary Rehren wedges which are Rehren duals to the boundary double-cones whose bases are the Ryu-Takayanagi complementary intervals.*



### Corollary

*Assuming the RT equality (and – say in $d = 1$ – assuming $c = 3/2G\sqrt{-\Lambda}$) the entanglement entropy of the Rehren bulk QFT between a pair of Rehren wedges is $1/4G$ times the length (in $d > 1$, area) of their shared ridge.*

This strongly suggests that the conventional belief [this is actually a near quote from Bianchi and Myers (2012)]

*In a theory of quantum gravity, for any sufficiently large region in a smooth background spacetime, the entanglement entropy between the degrees of freedom describing the given region with those describing its complement is finite and to leading order, takes the form* $S = A/4G$ *(plus lower order terms).*

should be replaced by . . .

…

*In a theory of quantum gravity, for any sufficiently large region in a smooth background spacetime, the entanglement entropy between the* **matter** *degrees of freedom describing the given region with those describing its complement is finite and to leading order, takes the form* $S = A/4G$ *(plus lower order terms).*

– and this is clearly nicely consistent with our proposed resolution to the Arnsdorf-Smolin puzzle, thus fulfilling our promise to give a further piece of evidence for that!