Free time evolution of a tracer particle coupled to a Fermi gas in the high-density limit

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Motivation

- Many results on derivation of mean field limits
- Show differences between Bosons on Fermions
- Fermionic mean field limits somewhat harder
- However: Mean field limits for a tracer in a filled Fermi-sea more stable

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Empirical data

1. Consider a (ideal) gas of Fermions or Bosons in the Ground state
2. A tracer particle interacts with gas
3. Bosons: Friction, Cherenkov radiation
4. Fermions: Free evolution if the tracer particle
5. Mean field works much better in fermionic case
6. Dirac sea

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The model

- $H = -\sum_{j=1}^{N} \Delta_{x_j} - \Delta_y + H_I$
- Interaction with tracer and gas particles: $H_I = \sum_{j=1}^{N} V(y, x_j)$
- No weak coupling!
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Mean field description

- Mean field is spatially constant
- Free time evolution
- Break down of mean-field description
- Marian von Smoluchowski:This is the same fallacy committed by a Hazard player thinking that he could never lose an amount larger than the stake of a single dice roll. Let us investigate this analogy further. [. . .] If one takes into account, however, that the particle with mass M undergoes 10^{16} such collisions in air, 10^{20} in water, most of which cancel each other with respect to the movement of the particle in X, but still produce a positive or negative excess of 10^8 or 10^{10}, then one would conclude that the particle would still suffer a change in velocity of about 10^2 or 10^4cm/sec.

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- High fluctuations in the forces $\sim \sqrt{\rho}$
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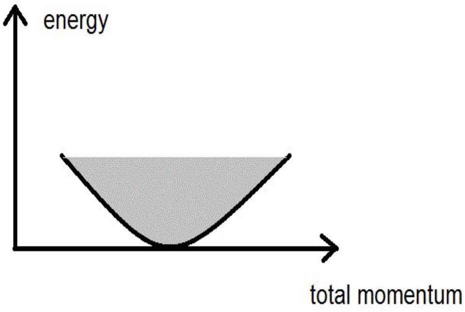
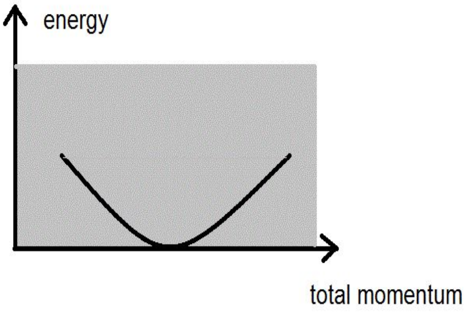
Fermions: one dimensional case

1d: easy: Momentum and energy conservation.



Fermions: $d > 1$

Higher dimensions



Theorem (Fermions: $d = 2$)

Let $V \in C_0^\infty$, $\chi_0 \in \mathcal{H}_y$ with $\|\nabla^4 \chi_0\| \leq C$ uniformly in ρ. Then, for any small enough $\varepsilon > 0$, there exists a positive constant C_ε such that

$$\lim_{\substack{N,L\to\infty \\ \rho=N/L^2=const.}} \left\| e^{-iHt}\Psi_0 - e^{-iH^{\text{mf}}t}\Psi_0 \right\| \leq C_\varepsilon (1+t)^{\frac{3}{2}} \rho^{-\frac{1}{8}+\varepsilon} \tag{1}$$

holds for all $t > 0$, where

$$H^{\text{mf}} = -\Delta_y - \sum_{i=1}^{N} \Delta_{x_i} + \rho \mathcal{F}[v](0) - E_{re}(\rho) \tag{2}$$

is the free Hamiltonian with constant mean field.
$E_{re}(\rho)$ is constant, subleading.

Estimate of fluctuations

- distinguishable particels: Fluctuation $Var(V) \sim \rho$
- Fermions: suppression due to fermi-pressure
- Calculations: exchange term gives negative contribution
- $Var(V) \sim \rho^{\frac{d-1}{d}}$
- Bosons: exchange term gives positive contribution
- Fluctuations roughly twice as large compared to distinguishable particels

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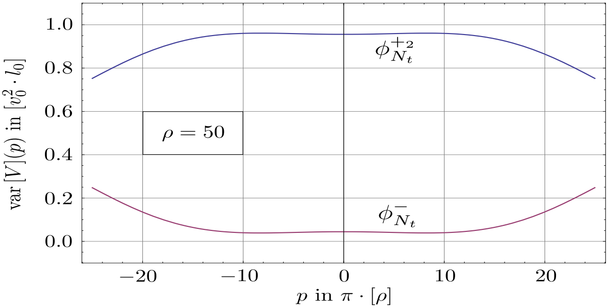
Estimate of fluctuations

Variance of force at some position y
(fermions: purble, bosons: blue)



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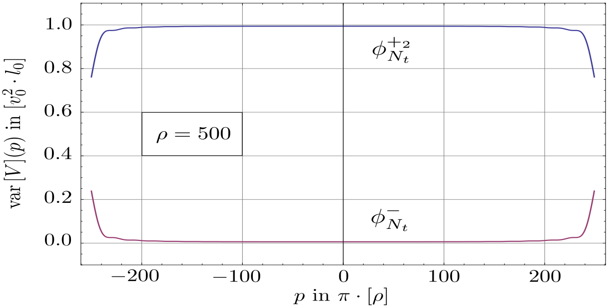

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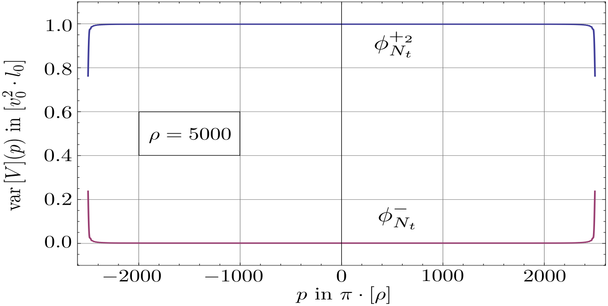

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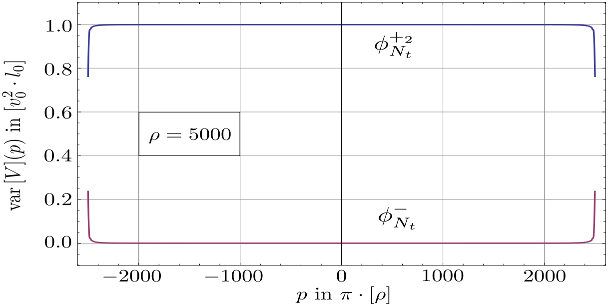

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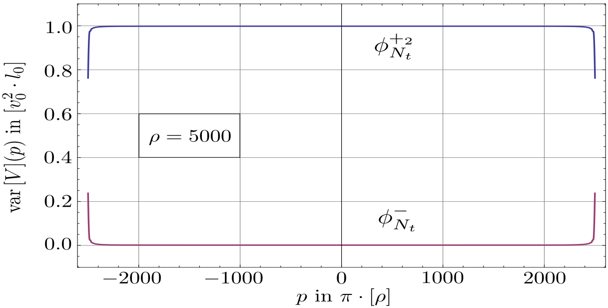

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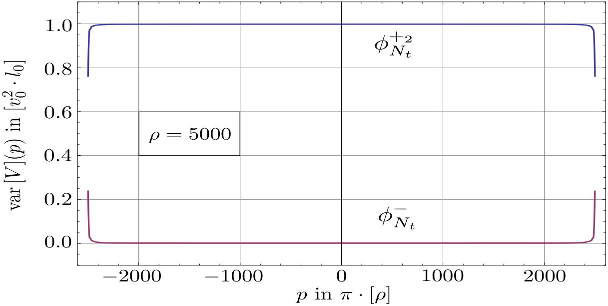

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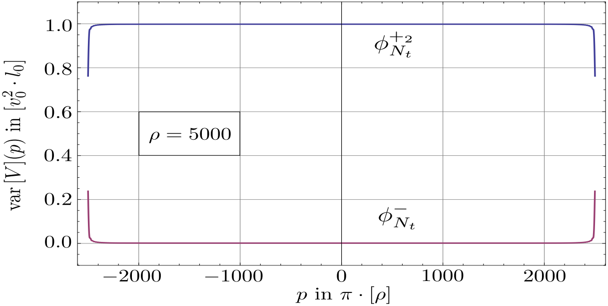

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- Stationary phase argument for other terms

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