Technische Universität München

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Sheet 8

Differential Topology: Exercise Sheet 8

Exercises (for Feb. 6th and 7th)

8.1 Define $f, g : \mathbb{R}^3 :\to \mathbb{R}$ as

$$f(x,y,z) = x^3y^2z^2 + y^4x - 7xz^2 \quad (1)$$
$$g(x,y,z) = 2x^3z^8 - 5y^2z + yx^4 \ . \quad (2)$$

Prove, using differential topology, that f and g have a common non-zero zero, i.e that there is an $a \in \mathbb{R}^3 \setminus \{0\}$ such that $f(a) = g(a) = 0$.

8.2 Let $f : S^n \to S^n$ be a smooth map that carries the antipodal points to antipodal points. Compute $\deg_2(f)$.

8.3 Construct a diffeomorphism $f : S^n \to S^n$ for which you prove that it is not smoothly homotopic to the identity map.

8.4 For any $n \in \mathbb{N}$ provide a smooth map $f : S^1 \to S^1$ with $\deg(f) = n$.

8.5 Let $f : S^n \to S^n$ be smooth and n even. Compute $\max_{x \in S^n} |\langle f(x), x\rangle|$ where x and $f(x)$ are regarded as unit vectors in $\mathbb{R}^{n+1}$.

8.6 Consider the following system of equations

$$2x + y + \sin(x+y) = 0 \quad (3)$$
$$x - 2y + \cos(x+y) = 0 \ . \quad (4)$$

Use the Euclidean degree to prove or disprove that there is a solution $(x_0, y_0) \in \mathbb{R}^2$ with $x_0^2 + y_0^2 < \frac{1}{4}$.