TECHNISCHE UNIVERSITÄT MÜNCHEN
Zentrum Mathematik

Prof. Dr. M. Wolf **Mathematical Introduction to Quantum Information Processing** Summer 2019
Blatt 9
http://www-m5.ma.tum.de/Allgemeines/MA5057_2019S (22.06.2019)

Read the sections *Information-disturbance* and *Time-energy* in the lecture notes!

1. *Complete positivity*
   Show that for the partial trace(s) positivity implies complete positive by using not much more than the definitions of the partial trace and of complete positivity.

2. *Quantum error correction*
   (a) Why are Pauli matrices used in the definition of an $[[n, k, d]]$-QECC? What if an 'error' occurs that is not described by one of the three Pauli matrices?
   (b) Assume you have encoded k qubits into n qubits using an $[[n, k, d]]$ quantum error correcting code. Unfortunately, $d - 1$ of the qubits were completely destroyed (a cat jumped out of a box and knocked over this part of the experiment). The good news is that the remaining qubits are perfectly intact. Show that and how you can perfectly recover the state of the original k qubits.

3. *Time-energy uncertainty relation*
   (a) Formulate and prove the Mandelstam-Tamm uncertainty relation for mixed states.
   (b) Consider a finite-dimensional Hamiltonian that satisfies $0 \leq H \leq \mathbb{1}$ and that governs the time evolution of a pure state via $\psi(t) = \exp[-iHt]\psi$. Let t_0 be the first time so that $\langle \psi, \psi(t) \rangle = 0$. Provide a lower bound on t_0 that is as good as possible and that does not depend on ψ.

4. Think about other points that you want to have discussed/clarified.