TECHNISCHE UNIVERSITÄT MÜNCHEN
Zentrum Mathematik

Prof. Dr. M. Wolf **Mathematical Introduction to Quantum Information Processing**

Summer 2019
Blatt 7
(07.06.2019)

http://www-m5.ma.tum.de/Allgemeines/MA5057_2019S

Read the section *Quantum channels and operations* in the lecture notes!

1. Derive the dual map (i.e., description in the Heisenberg picture) of the following quantum channels:
   (a) $T(\rho) := \lambda\rho + (1-\lambda)\mathrm{tr}[\rho]\,\sigma$, where σ is a density operator and $\lambda \in [0,1]$.
   (b) The partial trace $\mathrm{tr}_2 : \mathcal{B}_1(\mathcal{H}_1 \otimes \mathcal{H}_2) \to \mathcal{B}_1(\mathcal{H}_1)$.
   (c) $T(\rho) := \rho \otimes \sigma$ where σ is a density operator.
   (d) $T(\rho) := (\mathbb{1}\mathrm{tr}[\rho] + \rho^T)/(d-1)$, where $d < \infty$ is the dimension of the underlying Hilbert space.
2. Let $\sigma_1, \sigma_2 \in \mathbb{C}^{2\times 2}$ be the first two Pauli matrices. Give an explicit construction of two Hermitian, commuting block matrices Σ_1, Σ_2 that are such that σ_i is the north-west block of Σ_i, for both $i \in \{1,2\}$.
3. Prove the following statement: there is a Hilbert space $\mathcal{K}$, an isometry $V : \mathbb{C}^d \to \mathcal{K}$ and a Hermiticity-preserving linear map $R : \mathcal{B}(\mathbb{C}^d) \to \mathcal{B}(\mathcal{K})$ so that (i) $[R(\rho), R(\sigma)] = 0$ for all $\rho, \sigma \in \mathcal{B}(\mathbb{C}^d)$ and (ii) $V^* R(\rho) V = \rho$ for all $\rho \in \mathcal{B}(\mathbb{C}^d)$.
4. Which is the minimal number of Kraus operators necessary to represent the *phase damping channel* (see lecture notes for the definition)?
5. Think about other things you want to discuss (e.g. related to previous exercise sheets).