TECHNISCHE UNIVERSITÄT MÜNCHEN

Zentrum Mathematik

PROF. DR. M. WOLF **Mathematical Introduction to Quantum Information Processing** Summer 2019

Blatt 2

http://www-m5.ma.tum.de/Allgemeines/MA5057_2019S (03.05.2019)

Read the section *Bounded Operators* in the lecture notes!

1. Let $A, B \in \mathcal{B}(\mathcal{H})$ be Hermitian. Show that
   a) $\operatorname{tr}[AB] \in \mathbb{R}$ if $B \in \mathcal{B}_1(\mathcal{H})$,
   b) $A \geq B \wedge A \leq B$ implies $A = B$,
   c) $A \geq B$ implies that $CAC^* \geq CBC^*$ for all $C \in \mathcal{B}(\mathcal{H}, \tilde{\mathcal{H}})$.
2. Let $A, B \in \mathcal{B}(\mathcal{H})$ be positive and $B \in \mathcal{B}_1(\mathcal{H})$. Show that
   a) $\operatorname{tr}[AB] \geq 0$,
   b) $\operatorname{tr}[AB] = 0$ implies $AB = BA = 0$.
3. Let $P \in \mathcal{B}(\mathcal{H})$ be an orthogonal projection. Show that
   a) $0 \leq P \leq \mathbb{1}$,
   b) if $0 \leq A \leq \mu P$ for some $\mu \in \mathbb{R}_+$ and Hermitian $A \in \mathcal{B}(\mathcal{H})$, then $A = AP = PAP$.
4. For the operator norm on $\mathcal{B}(\mathcal{H})$, show that
   a) $0 \leq A \leq B$ implies that $\|A\| \leq \|B\|$,
   b) $-\mathbb{1} \leq C \leq \mathbb{1}$ iff $\|C\| \leq 1$ for Hermitian C.