① Does the following state-vector correspond to an entangled pure state (up to normalization)?

$|\psi\rangle = 2i|00\rangle - 4|10\rangle - |01\rangle - 2i|11\rangle$

② Let $\varrho \in \mathcal{B}(\mathbb{C}^2 \otimes \mathbb{C}^2)$ be the reduced density operator that is obtained after tracing out one of the qubits of a pure state given by $|\psi\rangle := \frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right)$. Is ϱ entangled?

③ Let $P_- \in \mathcal{B}(\mathbb{C}^d \otimes \mathbb{C}^d)$ be the projector onto the anti-symmetric subspace $\mathcal{H}_- := \{\psi \in \mathbb{C}^d \otimes \mathbb{C}^d \mid \mathbb{F}\psi = -\psi\}$. Prove that $\varrho := \frac{P_-}{\mathrm{tr}[P_-]}$ is entangled for all $d \geq 2$. (Hint: you may want to express ϱ in terms of $\mathbb{F}$)

④ For $d=2$ prove that for every density operator $\varrho \in \mathcal{B}(\mathbb{C}^d \otimes \mathbb{C}^d)$ is holds that:

$$(id \otimes \Phi)(\varrho) \geq 0 \iff (id \otimes \Theta)(\varrho) \geq 0$$

where $\Phi(X) := \mathrm{tr}[X]\mathbb{1} - X$ and $\Theta(X) := X^T$.

(Hint: you may want to use that the Pauli matrices σ_1 and σ_3 change sign when rotated like $\sigma_i \mapsto \sigma_2 \sigma_i \sigma_2$)