Quantum Effects [MA5047]

Sheet 6

Discussion: 21.01.2014

Exercise 1: Justify the first step in the main derivation of entanglement assisted teleportation, i.e. show that for all $B \in \mathbb{C}^{d \times d}$ it holds:

$$\mathrm{tr}[\rho_i B] = \frac{1}{p_i} \mathrm{tr}[(\Psi \otimes \omega)(M(j) \otimes B)]$$

where all the objects are defined as in the lecture notes.

Exercise 2: For a set $S \subseteq \mathcal{B}(\mathcal{H})$ of some Hilbert space $\mathcal{H}$, the *commutant* is defined as

$$S' := \{A \in \mathcal{B}(\mathcal{H}) | \forall X \in S : XA = AX\}$$

1. Let $\mathcal{H} = \mathbb{C}^d$ and S be the set of d^2 unitaries introduced in the lecture. Compute S'.
2. For the same set S determine the size of the smallest subset $S_0 \subseteq S$ such that $S_0' = S'$.
3. For $A \in \mathcal{C}^{d \times d}$ compute $\sum_{j \in \mathbb{Z}_d \otimes \mathbb{Z}_d} U_j A U_j^\dagger$ with the U_j as before.

Exercise 3: Assume Alice wants to send information to Bob by encoding it into the spin of an electron and then sending the electron (note that this is described within $\mathbb{C}^2$). Obviously, Alice can send one bit per electron. Show that, if Alice and Bob share entanglement (in the form of one maximally entangled state), then she can send two bits per electron.
What if a particle with larger spin is used (i.e. $\mathcal{H} = \mathbb{C}^d$, $d > 2$)?