Quantum Effects [MA5047]

Sheet 3

Discussion: 26.11.2013

Exercise 1: Let $\mathcal{H}$ be a finite dimensional Hilbert space and $A, B \in \mathcal{B}(\mathcal{H})$ arbitrary. Show

1. $\|A \oplus B\| = \max\{\|A\|, \|B\|\}$
2. $\|A \otimes B\| = \|A\| \cdot \|B\|$

where $\|.\|$ is the operator norm as usual.

Exercise 2: Let $\mathcal{H}_{A,B}$ be separable Hilbert spaces and consider the separable state $\rho := \sum_i \lambda_i \rho_A^{(i)} \otimes \rho_B^{(i)} \in \mathcal{S}_1(\mathcal{H}_A \otimes \mathcal{H}_B)$. Compute its reduced density operators.

Exercise 3: Let $\mathcal{H}_{A,B}$ be separable Hilbert spaces. Given $\rho \in \mathcal{S}_1(\mathcal{H}_A \otimes \mathcal{H}_B)$ such that ρ_A is pure, show that

$$\langle A \otimes B \rangle_\rho = \langle A \rangle_\rho \langle B \rangle_\rho \qquad \forall A \in \mathcal{B}(\mathcal{H}_A), B \in \mathcal{B}(\mathcal{H}_B)$$

Exercise 4: Let $\{e_i\}_{i=1}^d \subset \mathbb{C}^d$ be the standard basis. Then the maximally entangled state $\Omega \in \mathbb{C}^d \otimes \mathbb{C}^d$ is given by

$$\Omega = \sum_{i=1}^d \frac{1}{\sqrt{d}} e_i \otimes e_i$$

Show:

1. $(A \otimes \mathbb{1})\Omega = (\mathbb{1} \otimes A^T)\Omega$ for all $A \in \mathcal{B}(\mathbb{C}^d)$, where the transposition is in the standard basis.
2. Given $\psi \in \mathbb{C}^d \otimes \mathbb{C}^{d'}$, with $d' \geq d$, show that there exists $R \in \mathbb{C}^{d' \times d}$ such that $\psi = (\mathbb{1} \otimes R)\Omega$.