Prof. Dr. M. Wolf
M. Kech

Asymptotic Geometric Analysis

http://www-m5.ma.tum.de/Allgemeines/MA5024_2015W

Sommers. 2015
Blatt 5
(02.07.2015)

Hausaufgaben

5.1. Dvoretzky Dimension

a) Derive a non-trivial lower bound for the Dvoretzky dimension of l_p for $p \in (2, \infty]$.
HINT: Use the Dvoretzky criterion.

b) Show that for fixed $\epsilon > 0$ the Dvoretzky dimension of l_1^n is $k(l_1^n) = \Omega(n)$ as $n \to \infty$.

c) Let $X_n := (\mathbb{R}^n, \|\cdot\|^{(n)})$ be a sequence of metric spaces such that $k(X_n) := c(\epsilon)n^{1/4}$. Derive a non-trivial lower bound on the Dvoretsky dimension of the dual space X_n^*.

d) Formulate the geometric version of Dvoretsky's theorem. (In terms of ellipsoids, convex bodies, ...)

5.2. Dvoretzky-Rogers Theorem

Using the Dvoretzky-Rogers lemma from the lecture proof the Dvoretzky-Rogers theorem:

If a Banach space X is such that unconditional summability implies absolute summability, then $\dim X < \infty$.