



The following homework will be discussed on June 19th and June 22th:

**H9.1. (Neural networks - geometric interpretation)** Consider a feedforward neural network with activation function  $\sigma(z) = \mathbb{1}_{z \geq 0}$  and  $d$  inputs so that it represents a function of the form  $f : \mathbb{R}^d \rightarrow \{0, 1\}$ . Define  $A := f^{-1}(\{1\}) \subseteq \mathbb{R}^d$ .

- Assume the network has a single hidden layer with  $k$  neurons. What can you say about  $A$ ? (hint: recall which geometric objects correspond to single Perceptrons)
- Assume  $d = 2$ . Find a neural network architecture for which  $A$  can be the TUM logo.

**H9.2. (Neural networks - learning complexity)** Consider a feedforward neural network with  $d$  inputs, a single hidden layer with three neurons and a single output neuron. Assume all activation functions are  $\sigma(z) = \mathbb{1}_{z \geq 0}$  and that the output neuron has all weights and the threshold fixed so that it acts as  $x \mapsto \sigma(\sum_{i=1}^3 (x_i - 1))$ . Regarding the weights and threshold values of the three hidden neurons as free parameters gives rise to a function class  $\mathcal{F}_d \subseteq \{0, 1\}^{\mathbb{R}^d}$ .

- If  $f \in \mathcal{F}_d$ , what is the geometry of the set  $f^{-1}(\{1\})$ ?
- Consider a graph  $G = (V, E)$ . Look up (in the online or off-line source of your choice) what the 3-coloring problem is for  $G$ . Look what is known about its computational complexity.
- For any graph  $G = (V, E)$  define a “training data set”  $S \in (\{0, 1\}^{|V|} \times \{0, 1\})^n$  with  $n = |V| + |E| + 1$  as follows: denote by  $e_i \in \mathbb{R}^{|V|}$  the unit vector that has component 1 at the  $i^{\text{th}}$  position. Then  $S$  is assumed to contain the elements  $(e_i, 0)$  for all  $i = 1, \dots, |V|$ , the elements  $(e_i + e_j, 1)$  for each edge  $(i, j) \in E$  and the element  $(0, 1)$ . Show that  $G$  is 3-colorable if and only if there is a function  $h \in \mathcal{F}_{|V|}$  that correctly classifies  $S$ .
- What does this imply for the computational complexity of  $ERM$  for neural networks?