Open problems in quantum information theory
List of topics

Robert König and Michael Wolf
robert.koenig@tum.de m.wolf@tum.de

Abstract

We will focus on open problems in quantum information theory. These are connected to various modern topics, ranging from information measures, quantum communication and computation, to many-body quantum physics.

Format: The format of the seminar is as follows. Each participant gives a talk about a preselected topic and there will be plenty of room for discussions and questions concerning background and open questions. Talks will be prepared in advance. We will be available to assist with the preparation.

Prerequisites: Mathematical Introduction to Quantum Information Processing [MA5057] (or equivalent). A solid understanding of basic concepts of quantum information theory will be assumed.

Topics:

1. **Unitary quantum control:** Realizing unitaries by means of available physical operations is a basic primitive in quantum computation. A typical scenario concerns the case where parameters in a Hamiltonian can be controlled in a time-dependent fashion. Some algorithms for finding "control schedules" are known, but they are only efficient (polynomial-time in the Hilbert space dimension) under certain conjectures. [7]
2. **The no low-energy trivial states (NLTS) Conjecture:** Local Hamiltonians whose ground states cannot be prepared from a product state by a constant-depth circuit (or in constant time by local Hamiltonian evolution) are called topologically ordered. The NLTS conjecture posists that there are local Hamiltonians for which this property extends to all low-energy states: all states below a certain energy threshold are non-trivial to prepare. There is partial evidence for this conjecture, which connects to a number of open conjectures such as the quantum PCP theorem, and a conjecture about quantum low density parity check codes [2, 6, 3].
3. **Time warping with probability one:** Time evolution of a quantum system can be "reversed", changing the sign of the Hamiltonian and thus running time backwards. Surprisingly, this can be achieved with minimal control assumptions. However, currently known protocols only have a constant success probability. Whether this probability can be boosted to 1 remains open [9].

4. **The catalytic entropy conjecture:** Quantum information theory seeks to identify relevant information measures, often pursuing an operational approach. A new characterization of von Neumann entropy has recently be put forward. It relates entropy to "catalytic" convertibility of states by unitary operations: roughly, it states that $S(\rho) < S(\rho')$ if and only if ρ can be converted to ρ' with an ancillary system whose reduced density operator is left invariant. A weaker version of this characterization has been established, but the general case remains open.[1]

5. **Bell inequalities without input:** Quantum non-locality is "traditionally" expressed by Bell inequality violations. Recent work has uncovered a new form of quantum non-locality exhibited by distributions (rather than conditional distributions) generated by measurements of multipartite entangled states. Currently, a few striking examples are known. However, a more systematic understanding of such constraints would be desirable. [5, 10]

6. **Existence of non-trivial tensor stable positive maps** [8]

   Matrix transposition is, up to composition with completely positive maps, the only known map beside the identity that is positive and remains so after taking an arbitrary tensor power. Does this uniquely characterize matrix transposition among all linear maps on matrix space, or are there other maps that share this property? If there are others, this would have remarkable consequences for the distillable entanglement and the quantum channel capacity.

7. **The Markovianity problem** [11]

   The time-evolution of an open quantum system is often described in terms of a differential equation. Integrating the evolution up to some fixed time with underlying time-dependent Lindblad-generators leads to a quantum channel. It is, however, unknown which quantum channels can arise in this way.

8. **Zeroth law of thermodynamics** [4] Quantum systems in equilibrium are typically described by Gibbs states - states that maximize the entropy under given constraints. One way of deriving Gibbs states from operational assumptions is to demand dynamical stability of the system (i) itself, (ii) in contact with a second system and (iii) in contact with two or more systems. It is unknown whether the assumption (iii) is necessary for this argument.

References

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